

Chapter 11

The Multiple Origin of Blue Straggler Stars: Theory vs. Observations

Hagai B. Perets

11.1 Introduction

Blue straggler stars (BSSs) are stars that appear to be anomalously young compared to other stars of their population. In particular, BSSs lie along an extension of the main sequence (MS) in the colour-magnitude diagram, a region from which most of the stars of equal mass and age have already evolved. Such stars appear to be brighter and bluer than the turn-off point of the stellar population in which they were observed. Their location in the colour-magnitude diagram suggests that BSSs in old open clusters (OCs) and globular clusters (GCs) have typical masses of $1.2 - 1.5 M_{\odot}$, that are significantly larger than those of normal stars in such systems. Thus, they are thought to have increased their mass during their evolution. Several mechanisms have been proposed for their formation: *i*) stellar collisions due to dynamical interactions in dense stellar systems [24] *ii*) coalescence or mass transfer between two companions due to binary stellar evolution [50] *iii*) induced mergers/collisions through coupled dynamical/stellar evolution in triple systems [56]. The roles of each of these mechanisms in producing the observed BSS populations and their properties are still debated, as each of these scenarios were found to be successful in explaining some of the BSS observations, but fail in others.

In this review, we first discuss the observed properties of BSSs in the different environments (Sect. 11.2); we then describe the various models suggested for their formation (Sect. 11.3) and the long term evolution of BSSs in cluster environments (Sect. 11.4). Finally we compare the expectations from the different models with the known observable constraints and point out future theoretical and observational directions to advance the field (Sect. 11.5) and summarise (Sect. 11.6). Some of the subjects discussed in this review are explored in more details in other chapters of this book; and we refer the reader to these chapters when relevant. Though we discuss a wide variety of BSSs in different environments, our main discussion will

Physics department, Technion - Israel Institute of Technology, Haifa, Israel 32000,
e-mail: hperets@physics.technion.ac.il

focus on BSSs in old OCs and GCs which are best characterised; BSSs in other environments are discussed more briefly.

11.2 The Observed Properties of BSSs

Like any other stellar populations, BSSs are characterised through a wide variety of properties. These could be divided between intrinsic physical properties of the BSSs (mass, radii, composition, rotation, variability, temperature, luminosity); physical and orbital characteristics of multiple BSSs systems (binaries, triples); and the overall properties of the BSSs population (frequency, multiplicity, radial distribution). Another important division is between the directly observed BSS properties — e.g. colour-magnitude diagram (CMD) location — vs. inferred properties which require assumption dependent modeling (e.g. BSS mass). All of these properties may differ in different environments where BSSs are observed, and should therefore be discussed in the context of the relevant environment.

In the following we briefly discuss the observed properties of BSSs. Cases where the relevant properties are not yet well characterised/understood are specifically indicated in the table and text. An extended discussion about the observed properties of BSSs can be found in Chap. 3 and 5.

11.2.1 Physical Properties

11.2.1.1 Masses

The masses of single BSSs are not known, and can only be inferred through interpretation of their location in colour-magnitude diagrams as well as spectroscopic data, in the context of stellar evolution models. Detailed atmospheric models could potentially provide good constraints on the mass, and such models provide mass estimates of up to twice or more the turn-off mass for the brightest BSSs [60]. Stellar variability in SX Phe stars (all are BSSs in GCs) can also give various clues on the matter, and provide mass estimates up to twice and even three times the turn-off mass [53].

However, given the complex origin and stellar evolution of BSSs, and the inherent theoretical uncertainties in these modeling such interpretation might not be very reliable. In principle, the location of BSSs in the CMD shows them to be hotter and more luminous than stars on the main sequence, leading to the current interpretation of BSSs as stars more massive than the turn-off mass of their environment. More reliable methods make use of the dynamics of BSSs in multiple systems, where radial velocity measurements and/or eclipses can provide additional information. Even those methods can typically provide only partial data and/or constraints on the physical properties of a specific BSS. In cases where a BSS mass was determined

dynamically (in double-lined spectroscopic binaries), it was found that it was underestimated by 15% compared with the mass inferred from stellar evolution modeling of the CMD location ([20] and Chap. 3).

It is therefore premature to discuss a detailed mass function of BSSs. In the following we therefore refer only to the range of BSS mass inferred from the CMD, keeping in mind the potential large systematic deviations of these masses from the real BSS masses.

The CMD inferred mass function of BSSs in the OC NGC 188 [20] lies in the range of $1.15\text{--}1.55\text{ M}_{\odot}$ (see Chap. 3), i.e. $\Delta m = 0.15 - 0.55\text{ M}_{\odot}$ more massive than the cluster turn-off mass ($\sim 1\text{ M}_{\odot}$); a statistical estimate of the BSS masses based on orbital solution of binary BSSs in the cluster suggest a comparable but slightly lower mass range of $1.1\text{--}1.45\text{ M}_{\odot}$ [20]. Among field BSSs, Carney, Latham & Laird [5] find BSSs masses in the range $0.83\text{--}1.28\text{ M}_{\odot}$, i.e. $\Delta m = 0.03 - 0.48\text{ M}_{\odot}$ more than the turn-off mass (0.8 M_{\odot}). In other words BSSs can be significantly more massive than the turn-off mass, possibly requiring a large amount of mass accumulated onto them from an external source.

11.2.1.2 Rotation

The rotation velocities measured for BSSs extend over a wide range, showing both population of slow rotating and fast rotating stars (compared with the background population; see [43, 44] and Chap. 5 and 3), with varying distributions in different clusters. Systematic study of BSS rotational velocities in different environments is still in its infancy, and more data are needed before a clear interpretation of the data can be done (e.g., rotational velocity dependence on cluster properties). Relating these data to theoretical predictions is also premature, given the contradicting theoretical results regarding BSS rotational velocities (e.g., [4, 41]; see also Chap. 12). More theoretical as well as observational exploration is needed.

11.2.1.3 Composition

Though BSS composition could provide important constraints on their origin, e.g., showing pollution by accreted material from evolved stars, the available data is currently limited. We refer the reader to Chap. 5 as well as an overview by [44]. We will not discuss composition issues in this chapter.

11.2.2 Population Characteristics

11.2.2.1 Frequency

The overall frequency of BSSs in clusters is very small, but had typically been measured in detail only in GC cores and in open clusters. Typically, globular clusters contain a few, up to hundreds of BSSs, compared to the large numbers of stars in these clusters (few $10^5 - 10^6$ stars), providing BSS fractions of the order of a few $10^{-5} - 10^{-4}$.

Simulations of BSS formation in clusters (e.g., [26]) suggest that these fraction never become higher than these numbers even in the early evolution of a GC. A few tens of BSSs have been found in old open clusters such as M67 and NGC 188, providing a BSS fraction of a few 10^{-3} , i.e. much larger than that observed in GCs. Overall, it appears that BSS frequency is inversely proportional to the stellar density of the environment (for a more detailed discussion of these issues, see Chap. 9).

11.2.2.2 Multiplicity, Companion Type and Orbital Properties

Given the important role of binary or even triple companions in the suggested models for BSS formation, BSS multiplicity is one of their most important properties. Unfortunately, it is difficult to characterise. BSS multiplicity was studied in low mass field BSSs and in several open clusters, most notably M67 and NGC 188 [34, 21]; much less is known about the multiplicity of BSSs in GCs.

Field BSSs Carney et al. [5] used radial velocity measurements to study BSSs in the field and find their binary fraction to be high — consistent with all of the observed BSSs having companions. Their analysis shows BSSs binaries to typically have periods of 200–800 days, with low eccentricities compared with field binaries at the same period range, but distinctively not circular (see Fig. 11.1). Statistical analysis of the BSS companion masses show their mass to peak at $\sim 0.6 M_{\odot}$, with none of the BSS binary companions directly observed (all binaries were single-lined spectroscopic binaries). This was interpreted as pointing to white dwarf (WD) companions and therefore to a case C mass transfer scenario for the BSS formation.

OC BSSs The on-going effort to characterise the properties of BSS populations in OCs, have provided us with very detailed knowledge¹ about their multiplicity and the orbital properties of BSS multiples (see Chap. 3 for a detailed discussion). The BSS populations in M67 and NGC 188 show many similarities. In both clusters radial-velocity studies show the BSS binary fraction is much higher than the background stellar population ($60 \pm 24\%$ and $76 \pm 22\%$ for M67 and NGC 188, respectively), with the locus of the period distribution extending between 700 days and 3000 days. Though the upper limit is the observational limit, mostly due to the long time baseline required, the high binary fraction of BSSs even in this limited regime

¹ Currently, only for two OCs but on going study will provide similar data for additional OCs in the coming few years.

is much higher than that of field binaries [58], and points out to the important role of binaries. Few binary BSSs are found at shorter periods, with both cluster showing examples of peculiar double BSS binaries, and BSSs with more than twice the turn-off mass of the OC, indicating the need for many body (> 3) interaction origin. The eccentricity distribution is distinctively not circular and extending higher than that observed for field BSSs, but lower than the background binary population in the clusters ([20]; see Fig. 11.1). Statistical analysis of the companion mass shows it to peak at $0.55 M_{\odot}$ somewhat similar to the case of field BSSs.

No systematic study of BSS binarity in GC have ever been done. However, variability surveys of several GCs suggest that the frequency of eclipsing binaries among BSSs is much higher than that of field binaries [46, 59].

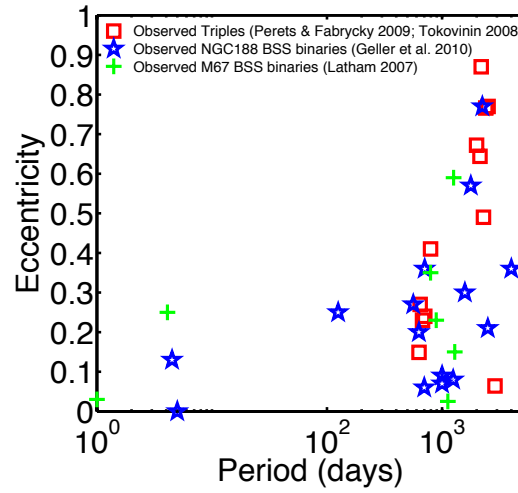


Fig. 11.1 The period-eccentricity distribution of BSS binaries in open clusters M67 and NGC 188, compared with the outer orbits or field triples with short period inner binaries.

11.2.2.3 Radial Distribution

Some of the early studies of BSS radial distributions in clusters have already shown them to be centrally concentrated (in OCs, see [47]; in GCs, see [3]). Later studies have revealed the existence of a bi-modal radial distribution in some GCs and OCs (for the OC NGC 188, see [21]; in GCs, see [17] and Chap. 5), with an inner centrally concentrated region followed by a dip in the distribution and a rise in the outer parts. The existence of a bi-modal distribution appears to depend on the cluster properties, and in particular the relaxation time, suggesting a major role of mass segregation in determining the BSS radial distribution. It is not clear whether other stellar populations show bi-modal radial distribution, though observations of eclipsing binaries suggest that short period binaries in the GCs ω Cen and 47 Tuc may give rise to such bi-modal distributions [63, 62, 56].

11.2.2.4 Multiple Populations in GCs

Recently, it was shown that some GCs appear to host two distinct populations of BSSs as observed in the CMD. The origin of such multiple populations is yet to be studied in detail. We refer the reader to Chap. 5 for further discussion of this issue.

11.3 Models for Blue Straggler Star Formation

All current models for BSS formation are based on the assumption that these stars are rejuvenated through mass transfer. The difference between the models are the type of processes leading to such additional mass accumulation. All models require an external source of material and can be generally divided into three classes of mass transfer: merger, collision or accretion. Mergers occur when two MS stars, typically in a short period binary, come into contact and eventually merge together to form a more massive star containing most or all of the the mass of the mergers binary. Collisions of two MS stars are a more violent scheme where two MS stars form a merged star through a fast dynamical encounter; these could occur through the collision of two unrelated stars in a dense cluster, or possibly in unstable triple stars where two stellar companions collide. The more violent nature of these events could produce different outcomes than the more gentle merger processes, and can potentially produce BSS with different physical properties. Finally, stellar companions could shed mass through winds or Roche lobe overflow and the ensuing accretion can then rejuvenate the stars to become BSSs.

There is a wide variety of stellar systems and different types of evolutionary processes that could lead to these mass exchange scenarios; in the following we discuss these models in more detail.

11.3.1 Collisions in Dense Clusters

Early on, stellar collisions in dense clusters have been suggested as a channel for BSSs formation. This channel could play a major role in BSS formation in GCs (e.g. [7, 26], and references therein) and may contribute to the BSS population in the cores of OCs [25]; although it is not likely to serve as the dominant formation channel in OCs [38, 25, 56, 19]. Obviously, this channel is irrelevant for field BSSs, where physical collisions are extremely rare. This formation channel is discussed in more details in Chap. 9; here we provide a brief discussion and focus on the issue of binary BSSs in this context.

Collisional merger of stars is very efficient and conserves most of the mass of both merged stars for low velocity encounters as expected in OCs and GCs ([4]; large mass loss could occur at impact velocities at infinity comparable to the escape

velocity from the stars), allowing them to form BSSs of up to twice the turn-off mass of the cluster (or even more, if more than two stars collide).

The rate of stellar collisions is strongly dependent on the number density of stars in the cluster. Collisions can occur through the direct physical collisions between single stars in the cluster, but encounters between higher multiplicity systems are more likely to mediate most physical collisions in dense environments [37, 18, 55, 36, 7].

It was therefore expected that a strong correlation between the collisional parameter in GCs (see Chap. 9) and the specific frequency of BSSs should exist. A correlation with the binary fraction, given their role as collision mediators should also be apparent. However, though observations do show a correlation with the GC binary fraction, the strongest correlation is found to be with the GC mass, while no correlation is found with the calculated collisional parameter (see [35] and references therein). Most interestingly, Chatterjee et al. [7] have recently made detailed simulations of the evolution of GCs in their BSSs populations, and found that binary mediated stellar collisions are the dominant channel for BSSs formation in dense clusters. Moreover, they find a clear, though weak correlation with the cluster collisional parameter. They suggest that the calculated collisional parameter based on observational analysis of GC properties is inaccurate, due to accumulated errors in the various observational parameters; in fact, they find no correlation between the intrinsic accurate collisional parameter calculated for their simulated GCs and an “observed” collisional parameter obtained by making use of an observational-like analysis of the cluster properties. These results may explain the conundrum in correlation between the “observed” GC collisional properties and the BSS population which was debated over the last few years and can provide for various important pointers for new observations.

Binary-single and binary-binary encounters are very likely to leave behind a binary BSS; Chatterjee et al. [7] find that $\sim 60\%$ of the BSSs in their simulations are in binaries. Studies of binary-binary and binary-single encounters [39, 40, 10, 18] show that binary-binary encounters leave behind BSS with long period binary companions (most typical are at periods of $10^3 - 10^4$ days) with an almost thermal eccentricity distribution (average eccentricity of $2/3$ and somewhat lower for the shorter period binaries). Detailed hybrid Monte-Carlo models coupled with few-body simulations of GCs also account for the later evolution of the binaries, and show the binary distribution peak at a few tenth to a few astronomical units; only a small fraction ($< 10\%$) have small semi-major axis comparable with typical eclipsing binaries found in GCs.

Finally, we note that none of the studies of GC evolution have accounted for primordial triples and their evolution. In addition, though dynamically formed triples have been shown to form quite frequently in GCs, and potentially play a non-negligible role in BSS formation [27], their long term evolution in GCs have not been studied. Given the potentially important role of triples in mediating BSS formation [27, 56, 36], this is an important direction for future theoretical studies of BSSs and other exotica in GCs.

11.3.2 *Binary Evolution*

Binary stellar evolution (BSSE) was one of the first models suggested for the origin of BSSs [51]. This general term refers to several possible scenarios and outcomes. In the BSSE model, the evolution of a stellar binary leads to mass transfer from a star to its binary companion, thereby increasing its mass, potentially a long time after its formation. A stellar binary could merge in which case the final product will include most of or all the mass of both companions; alternatively the companion might shed mass through Roche-lobe overflow (RLOF; see Chap. 8), winds or wind-RLOF (see Chap. 7) thereby transferring part, or most of its envelope to the now rejuvenated primary, leaving behind a white dwarf. Various aspects/sub-channels for the BSSE model for the formation for BSSs are discussed in various other chapters in this volume (Chap. 7, 8, 9, and 12); here we provide a general overview for these the various sub-channels, their differences and their implications in a more general context. When applicable we will refer the reader to the relevant detailed discussions in other chapters.

Mass transfer during BSSE is traditionally divided into three categories; depending on the state of evolution of the mass donor interior [30]: MT during the MS (case A), beyond the MS but before helium ignition (case B), or beyond helium ignition (case C). Eggleton [12] refines case A MT into many more sub-categories (not detailed here; we refer the reader to [12]), and redefines case B and C MT: Case B is the situation where the mass donor is in the Hertzsprung gap, with a mainly radiative atmosphere, and case C is the situation where the mass donor is on the giant branch, and therefore have a mainly convective envelope. We will not discuss these scenarios in depth, but rather remark on their main implications for BSS and its companion, and focus only on BSS formation during MT.

MT scenario are generally thought to lead to orbital circularisation. However, binary systems thought to be produced through MT many time show distinctively larger eccentricities (see Fig. 11.2), suggesting that our understanding of this process is incomplete. Some studies suggests scenarios where higher eccentricities are kept (see [11] and references therein).

Case A MT: In order for a MT to occur during the MS, the initial binary separation must be small enough for the RLOF to ensue (at most a few solar radii), given the compact radius of the companion MS star. Various processes could lead to such a close configuration, involving magnetic braking, tidal evolution or possibly also affected by perturbations from a third companion in a triple (see Sec. 11.3.3 for the latter). In dense stellar clusters, interactions with other stars in the cluster may also results in shortening the binary period, exciting the binary eccentricity (thereby leading to a smaller peri-centre approach where effects from the the first processes can become significant), or even exchange of companions to produce a new shorter period binary.

RLOF on the MS will increase the gainer mass, making it a BSS. Such evolution could lead to evolution into contact configuration (during the evolution they system could get in and out of contact, depending on the specific case), and possibly merge. A merged system will form a massive BSS, an unmerged system would form a

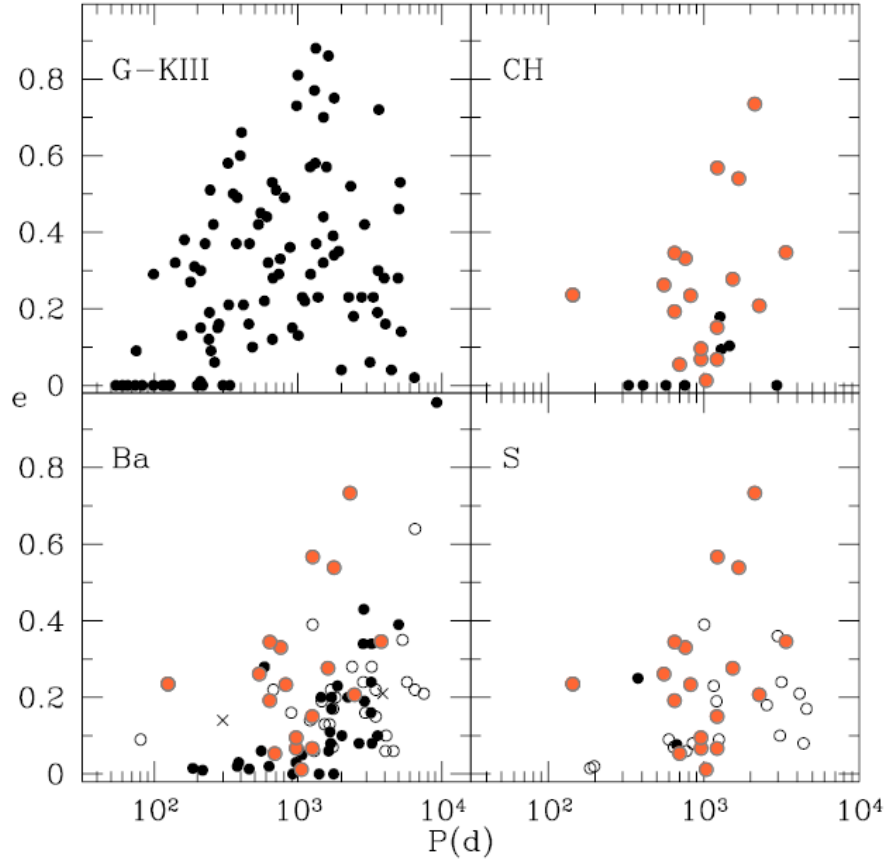


Fig. 11.2 The period-eccentricity distribution of several populations of evolved binaries, compared with the BSS binaries in M67 and NGC 188 (adapted from [28]). Upper left panel: Binaries involving G and K giants in open clusters [52]; Upper right panel: CH stars [49]; Lower right panel: S stars [28]. BSS binaries in the open clusters M67 and NGC 188 are shown as large (orange) filled circles. Note that although BSSs were likely to accrete more mass than the various type of polluted stars, their eccentricities are much higher than those of the polluted star binaries, even at short period of a few hundred days (the most comparable are the Ba stars; however, these are stars far more massive than the BSS binaries, whereas the CH stars with more comparable masses show significantly lower eccentricities. In other words, it appears that mass transfer does induce circularisation of binaries (though less efficiently than the typical theories suggest), as evident from the lower eccentricities observed for the polluted stars. The origin of the higher eccentricities at low periods for the BSS binaries (which accrete more mass than polluted stars) is therefore inconsistent with the overall period-eccentricity distribution of OC BSSs, suggesting that at most only a small fraction of them can be explained by case C mass transfer. Mass transfer may still explain the origin of the lowest eccentricity BSS binaries observed; whereas the rest of the BSS binaries might be explained by the triple origin; see Fig. 11.1 for comparison of period-eccentricity distribution in triples.

BSS with a short period companion, which could be long lived (1–2 Gyr); contact configuration might be observed as W UMa type eclipsing binaries. Case A MT therefore lead to either single massive (on average, given the complete MT) BSS, or to a less massive (on average, given the only partial MT) BSS with a short period hydrogen burning companion (note however that the companion could be affected by the interaction, its appearance not necessarily resembling a MS star).

Case B MT: In this case the evolution into RLOF occurs only following the companion evolution to Hertzsprung gap, in longer period binaries (few to tens solar radii). For significant accretion to occur leading to BSS formation, the system should not go through a common envelope stage which will eject the envelope rather than lead to mass growth. Therefore, scenarios leading to a BSS formation will leave behind a BSS with an intermediate period (few $\times 1 - 10$ days) and a helium WD companion, following the mass donor stripping; see [33] for a detailed example of such a scenario used to explain the origin of the 1040S system in the OC M67. In principle the BSS binary could also be observed during the accretion phase, but only for a relatively short time of a few hundred Myr. The BSS could be quite massive, though 0.1–0.4 M_{\odot} of the final system mass will reside in the helium WD. Such BSSs are not likely to be observed as eclipsing binaries (due to the small radius of the WD companion, as well as the expected wide separation), unless observed during the accretion phase.

Case C MT: In this case the donor star is already quite evolved, with a large radius. The initial binary period is therefore expected to be in the range of tens to a few hundreds or even 1–2 thousand days. In systems with up to a few hundred days period, a BSS could be formed through accretion, with up to $\Delta M \sim 0.2 M_{\odot}$ above the turn-off mass. Stellar evolution calculations (e.g. [8]) suggest that binaries with larger periods can only produce BSSs very close to the turn-off mass, which are not considered as BSSs in current observational criteria. After its evolution the donor star will become a CO WD, with typical mass of 0.6 M_{\odot} .

Case D MT (wind RLOF): Binaries with wide separations (typically > 4 AU) will not evolve through RLOF, as the primary star do not fill its Roche-lobe. MT could still occur through the accretion of slow wind material ejected from an asymptotic giant branch (AGB) star. Simple calculations using Bondi-Hoyle accretion model suggest such accretion is inefficient at large separations ($< 15\%$). However, recent hydrodynamical simulations showed that at binary periods of $\sim 2,000 - 10,000$ days, the wind can be focused by the accreting star and the accretion efficiency can be as high as 45% [1]. It is therefore potentially possible to form even massive BSS ($\Delta M > 0.4$) through this process. The leftover companions should be CO WDs, following the regular evolution of the donor star after the AGB.

11.3.3 Triple Evolution

Binary stellar evolution has been suggested early on as a channel for BSS formation through MT and mergers. In recent years, however, it was realised that triple

stars can have an important, and sometime major role in affecting binary evolution. Interestingly, the evolution of triple system could mediate the production of BSSs through various different channels thereby allowing for the formation of BSSs through mergers, MT and even collisions. In the following we discuss these various channels, focusing mostly on secular evolution through Kozai-Lidov cycles, coupled with tidal friction (KCTF).

11.3.3.1 Secular Evolution Coupled with Tidal Friction

As discussed above, one of the channels for BSS formation is case A MT, through which stars in short period binary systems can transfer mass or even merge on the MS, thereby leading to the formation of massive BSSs. Such short period binaries can not easily form following the evolution of pre-MS binary, since the pre-MS radius of stars can sometime be larger than the short period binary separation. It was therefore suggested that the formation of short period binaries is mediated by triple dynamical evolution [31, 13, 14, 15] coupled with tidal friction processes, as we shall discuss in the following. Following this, Perets & Fabrycky [56] suggested that triple stars could serve as natural progenitors for BSSs, and in particular could explain the existence of eccentric and wide orbit BSS binaries observed in OCs, as triples in which the inner binary have merged, leave behind a BSS with a long period companion.

Stable triple systems require a hierarchical configuration in which two stars orbit each other in a tight “inner binary”, and the third star and the inner binary orbit their common centre of mass as a wider “outer binary”. Such triples are long lived, but secular evolution can change their orbital inclination and eccentricity. A particularly important change was discovered by Kozai [32] and Lidov [42] in the context of solar system triples (Sun-asteroid-Jupiter or Sun-Earth-satellite). They found that if the inner binary initial inclination relative to the outer binary orbit is high enough, secular torques will cause its eccentricity and inclination to fluctuate out of phase with one another, leading to periodic high amplitude oscillations in the inclination and eccentricities of the triple inner binary; these are typically termed “Kozai oscillations”. Lidov [42] noted that the large oscillations in the amplitude of the inner binary eccentricity might even lead to collision between the inner binary members. Collisions were prominent also in the first application of these dynamical concepts to triple stars. Harrington [23] noted that large initial inclination ($i_c \lesssim i \lesssim 180^\circ - i_c$, for a “Kozai critical angle” of $i_c \approx 40^\circ$), leads to large eccentricities, which could cause a tidal interaction, mass loss, or even collision of the members of the inner binary. Thus, Harrington [23] reasoned that a triple star system with an inner binary mutually perpendicular to the outer binary should not exist for many secular timescales. However, as noted by [48], the inner binary stars, while coming close to one another, will not merge immediately; instead, the tidal dissipation between them shortens the semi-major axis of the inner binary during these eccentricity cycles. They suggested that such inner binaries could therefore attain a very close configuration, in which mass transfer and accretion could occur, possibly forming

cataclysmic variables or binary X-ray sources. Kiseleva, Eggleton & Mikkola [31] were the first to show that coupling of tidal friction to the high amplitude Kozai-Lidov secular evolution could be a highly efficient mechanism for the formation of short period binaries, as was later studied by [14, 15]; we refer to this mechanism as Kozai cycles and tidal friction (KCTF; note that the last stages of the merger are likely to be induced by magnetic braking).

Observations of short period binaries showed that more than 90% of short period F/G/K binaries with periods of $P < 3$ days — consistent with 100%, when considering completeness — have a third companion. The fraction gradually decreases to $\sim 30\%$ at $6 \leq P \leq 30$ days, consistent with the overall background level ($\sim 30\%$ of all F/G/K binaries are triples [58]). These observations suggest that all short period binaries ($P < 3 - 6$ days) form in triple systems, giving credence to the KCTF formation scenario.

Perets & Fabrycky [56] have taken the next logical step; if short period binaries are formed through KCTF evolution in triples, and short period binaries are typical progenitors of BSSs, then triples could be the natural progenitors of BSSs. In Fig. 11.3 we show an example for the KCTF scenario and the formation of short period binaries that will later merge.

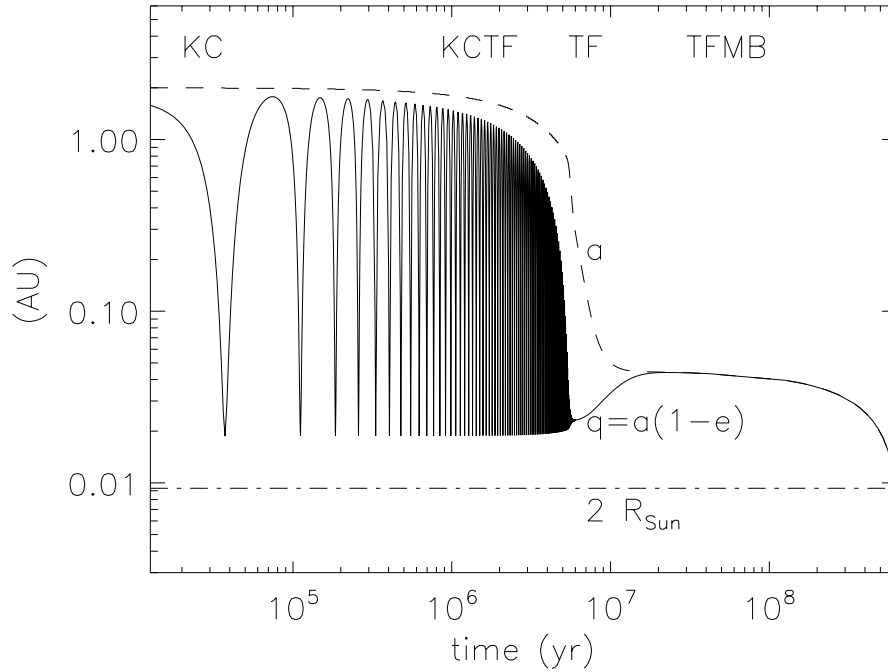


Fig. 11.3 Merger of the two stars of an inner binary, accomplished by a combination of Kozai cycles, tidal friction, and magnetic braking. (Reproduced from [56] by permission of the AAS)

Most interesting, this basic scenario provides a wealth of unique observable predictions. These could be summarised by the following:

1. Merged short period binaries in triples will leave behind a BSS and a binary (originally the third) companion. Therefore the binary frequency of BSSs should be high, with typically wide periods, otherwise the original triple would not have been stable.
2. BSSs in short period binaries should still have a third companion on a wider orbit.
3. The BSS mass (and luminosity) can be high, and as much as twice the turn-off mass in the case a full merger of the inner binary; this is to be compared with BSSs in wide orbit binaries from case B/C MT, where a significant mass is left in the WD companion, and the MT efficiency is not expected to be high (typically at most $0.2 M_{\odot}$ can be transferred to the mass accretor).
4. The period-eccentricity diagram of BSS binaries should show strong similarities to the period-eccentricity diagram of the **outer** binaries of triples with inner short period binaries. Such a comparison is shown in Fig. 11.1; it appears that such binaries should have long periods ($\gtrsim 500$ days) and eccentricity distribution comparable to that of field binaries with similar periods.
5. The wide orbit companion of BSSs could be a MS star in contrast with case C or case B MT which might produce a wide orbit BSS binary, but the companion in that case must be a CO (typical mass of $0.5\text{--}0.6 M_{\odot}$; case C) or a helium white dwarf (of $0.15\text{--}0.45 M_{\odot}$; case B).

Recent studies have shown that inclined triple systems with smaller hierarchies (i.e. the outer to inner period ratio is small, of the order of a few times the critical ratio for the system stability), show quasi-periodic oscillations, similar to Kozai cycles, but more chaotic, and which can lead to large eccentricity change on dynamical timescales, and to higher eccentricities ([2, 29], see also [22] for a related study). Such behaviour extends the Kozai-induced mergers to the small hierarchy regime, and could lead to direct collisions even without significant tidal evolution, i.e. triple dynamical evolution could lead to physical collisions and not only mergers/case A MT.

11.3.3.2 Collisions in Destabilised Triples

Recently, [57] have suggested that stellar evolution and mass loss from the inner binaries of triples can lead to the triple destabilisation and the occurrence of collisions and close tidal encounters between the triple components. Such triple evolution could therefore lead to the formation of BSS when two MS stars collide. However, it is not clear that such evolution could produce a significant population of BSSs, and more detailed triple stellar evolution studies are needed (see also [22] for a population synthesis calculations, but limited to wide binaries of $a > 12$ AU; much beyond observed binary BSS separations in OCs).

11.3.3.3 Accretion onto a Binary from a Third Companion

Another interesting evolutionary scenario is the case where a third companion in a triple evolves and sheds its mass on the inner binary. Such a scenario has not yet been studied in details. In principle, the transferred mass might form a common envelope around the inner binary, leading to its in-spiral and the formation of a short period binary that could later evolve to form a BSS through a case A MT. In addition, or alternatively, the mass can accrete onto the inner binary components potentially rejuvenating both of them and forming a double BSS binary with a WD third companion on a wide orbit. Whether these scenarios are physically plausible is yet to be confirmed in detailed studies (see also Chap. 4).

11.4 Long Term Dynamical Evolution of BSSs in Clusters

11.4.1 Mass Segregation in Clusters

BSSs are more luminous and bluer than the background population in which they are detected, suggesting their mass to be higher than the turn-off mass. In particular, this would make BSSs the most massive stars in stellar clusters, beside neutron stars and black holes. Dynamical friction (see Chap. 10 and 9) leads to the segregation of the more massive stars into more centrally concentrated distribution compared with the background population of lower mass stars. Currently observed evolved stars (red giants and horizontal branch (HB) stars) have evolved off the MS only relatively recently, i.e. their mass during most of their evolution was close to the currently observed turn-off mass of the cluster, and they were among the long lived most massive stars in the cluster. Since these stars are also more massive than the typical stars in the cluster they should also be mass segregated, though slightly less than BSSs.

BSS radial distribution in clusters is typically compared with that of HB or red giant stars. However, given the relatively small difference in mass between this background population and the BSSs, it might be surprising that BSSs appear to be more segregated than these populations. In particular, Mapelli et al. [45] and, more recently, Ferraro et al. [16] have suggested that the bimodal radial distribution in clusters is due to the segregation of the BSSs compared with the HB stellar population. They provided a simplified formula for the position in the cluster at which the timescale for BSS mass segregation is comparable to the cluster lifetime, and showed that it appears to be consistent with the trough in the bimodal distribution of BSSs, showing this as evidence for this process. However, if one were to use the same approach on red giant stars, taking their mass to be slightly smaller than the turn-off mass of the cluster (as expected for most of their evolution), one finds that they should similarly show a trough not far from the location of the expected BSS trough. In other words a relative comparison of the two populations should not have

shown a significant trough in the relative populations just due to mass segregation. The simple theoretical interpretation therefore appears to be discrepant with the observations. This might arise from the various assumptions made. For example, the assumed mass of the BSSs was taken to be $1.2 M_{\odot}$, however, BSSs might have a much higher binary fraction than that of the background population, consistent with several of the BSS formation channels. BSSs are therefore likely to have a binary companion, and the mass of the BSS system should therefore typically be the mass of a binary rather than the assumed mass of a single BSS. This might remedy the discrepancy described above, and not less important point out that the observed GC BSSs are likely to be binaries; an important clue about their origin. Other problem may arise from not accounting for the different lifetimes of the BSS population and that of the compared population (M. Giersz, priv. comm. 2012).

11.4.2 Dynamical Evolution of BSSs Binaries

BSSs can form through one of the evolutionary channels discussed above, leaving them either as single stars or in a multiple system, which could later change. In clusters their initial configuration could dynamically evolve through encounters with other stars in the clusters. Hurley et al. [25] provide a detailed analysis of such later dynamical evolution observed in simulations of an open cluster, and present the diverse possibilities and outcomes of such evolution. The complicated dynamical evolution of binaries in cluster is beyond the scope of this review, but we will briefly discuss some of the main results relating to BSSs. Encounter between multiple systems typically lead to the more massive stars residing in the binary and the least massive stars being ejected. BSSs are the most massive stars in their host cluster and therefore have a higher probability to be exchanged into binaries, even if they originally formed as single BSSs. Detailed analysis of the evolution of open and globular clusters showed that the BSS binary fraction in these simulations were high, mostly due to their formation channels, but their later dynamical evolution kept them in binaries, and the majority of BSSs are found in binaries at the end of the simulations [25, 7, 26].

Dynamical encounters between binary and single stars typically leave behind relatively eccentric binaries. Hurley et al. [25] find the period-eccentricity diagram of BSS binaries in their simulation to be inconsistent with observations. In particular, most of the wide binaries of 200–700 day period were typically formed through dynamical encounters, leaving behind highly eccentric binaries, whereas most observed BSS binaries reside in larger periods (> 500 days) and much less eccentric orbits (see [56] and [20]).

11.5 Blue Straggler Stars: Observations vs. Theory

The various theoretical expectations and observational results discussed above are summarised in table 11.1 and 11.2. In table 11.1 we show a summary of the observed properties of BSSs in different environments, to be compared with the summarised theoretical predictions shown in table 11.2. The latter include single processes, as well as results of large simulations of GCs [7, 26] and OCs [25, 19] which include both dynamics and stellar evolution, but do not include triple stars.

In the following we briefly discuss the theoretical expectations vis a vis observations in the different environments.

11.5.1 Globular Clusters

Recently extensive Monte-Carlo simulations of GC evolution [7, 26] provided for the first time detailed predictions regarding the population of BSSs in these environments. These simulations include simplified stellar evolution prescriptions and detailed account of binary dynamics and interactions. These simulations do not account for primordial or dynamically formed triples. The detailed simulations provide the overall evolution of the BSS population in GCs, but here we will focus on the final outcome at the typical age of observed GCs (~ 12 Gyr). The simulations suggest that most of the currently observed BSSs today result from direct physical collisions during binary-single and binary-binary encounters. The total number of BSSs in the simulations is consistent with the observed numbers of BSSs in GCs. The BSSs are centrally concentrated, but can show a bi-modal radial distribution. The majority ($\sim 60\%$) are in binaries with a wide orbital distribution between a few days and a few thousand days, but mostly distributed at $\sim 100 - 1000$ days, with a small fraction ($\sim 10\%$) at short periods. We conclude that these results are generally consistent with the observed known properties of GC BSSs, and can explain the correlation between the binary fraction and the BSS fraction (mostly due to the dominance of collisions in binary encounters). Unfortunately, detailed knowledge of the BSS properties such as exist for a few open clusters precludes more detailed comparison (e.g., with BSS binary frequency and binary orbital properties). It is also not yet clear whether these models can explain the observed correlation of the normalised BSS fraction with the mass of the GCs [35]. In addition, the existence of a significant fraction of observed eclipsing BSSs, as discussed above, suggest a large fraction of the BSSs are in short period binaries with MS companions, inconsistent with the theoretical models. Such short period BSS binaries may arise in the triple evolution scenario where the induced formation of a short period inner binary leads to case A MT, and the BSSs to be observed as eclipsing binaries. However, at this point the data of eclipsing binaries is very sparse and detailed dedicated observational study of GC eclipsing binaries as well as theoretical study on the role of triples in GCs is required in order to resolve this issue.

11.5.2 Open Clusters

Detailed hybrid N -body/stellar evolution simulations by [25] and [20] were done for old OCs similar to M67 and NGC 188 for which detailed data observations exist. A detailed overview of the observations and simulation results can be found in Chap. 3. Overall they find that the number of BSSs in the simulation is less than a third of that observed and they can not reproduce the bimodal radial distribution of BSSs observed in NGC 188; moreover the BSSs in the simulations are much more centrally concentrated than the observed ones. The BSS binary fraction in the simulations is only $\sim 15\%$, but a fifth of the observed value. To quote these authors

the deficiency in number of BSSs, the low frequency of detectable binaries among those that are formed, and the lack of a bimodal BSS radial distribution are striking failures of the model compared with the observations.

Nevertheless, Mathieu and Geller suggest that at least the orbital properties of the BSS binaries in the simulations are consistent with the observed BSS binary properties, reproducing the long periods of the BSSs, non-circular orbits and companion mass distribution. Since most of these binaries arise from case C MT scenario, they conclude that this is the likely main mechanism for the BSS production in OCs, irrespective of the other failures that might be remedied with better stellar evolution models. However, case C MT that lead to the long orbital periods with high eccentricity is highly inefficient in transferring mass, producing BSSs with typical $\Delta M \approx 0.1 M_{\odot}$, smaller than typically inferred from the CMD location of the BSSs. We therefore conclude that case C MT may explain a fraction of the observed BSSs, but it is difficult to see how it can explain the majority of the OC BSSs. That being said, our current understanding of MT in binaries is very limited, and future studies may show that more efficient MT can occur (e.g., some form of case D MT). Evolution of primordial triples may help produce the observed binary BSSs, not only naturally explaining the long orbital periods and high eccentricities (see Fig. 11.1), but also explaining the very high binary fraction, and the high mass of the BSS inferred from the CMD. Detailed cluster simulations which include significant fraction of triples and detailed accounting for KCTF are needed in order to check whether a significant number of BSSs can indeed form in this way.

Figure 11.4 shows the outcome of population synthesis of evolving binaries resulting in mergers in case A MT, compared with case C MT. Case C MT can explain the wide orbits of the BSS binaries in NGC 188, but the resulting BSSs are too faint and cannot explain the observed BSS population. The CMD location of full mergers is consistent with the observed BSS population, but then a third companion must be invoked to explain the binarity of the observed BSS populations.

BSS formed through the case D MT could easily have wide orbit companions, and even their eccentricity needs not be affected much. However, the expected periods for this scenario are higher than the typical periods observed for BSS binaries in OCs.

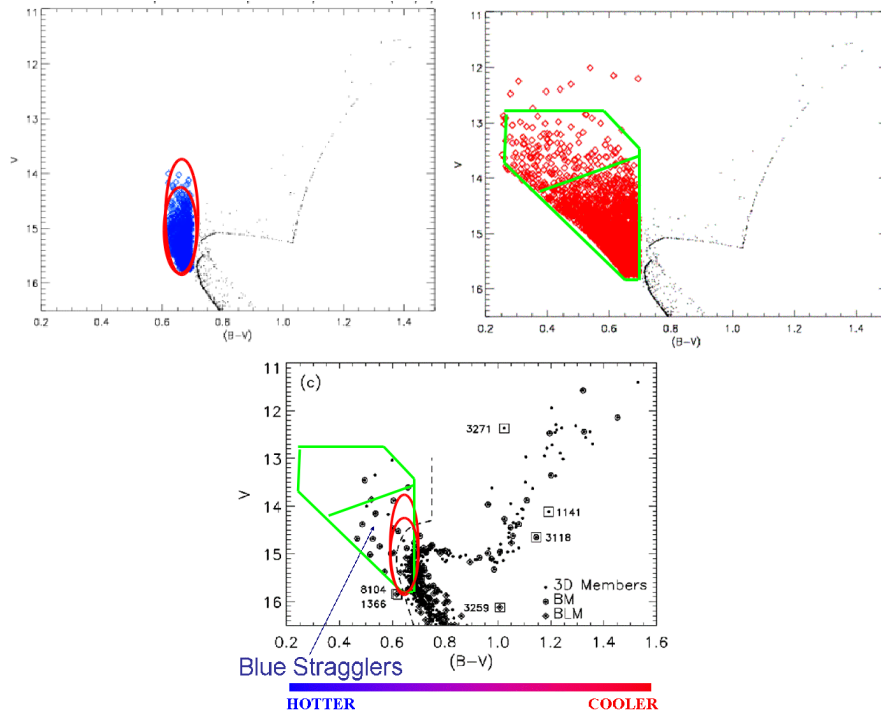


Fig. 11.4 Position of NGC 188 BSSs on the CMD. Top left: Population synthesis results of BSSs formed through case C mass transfer. Top right: Population synthesis results of BSSs formed through mergers. Bottom: The location of observed BSSs in the open cluster NGC 188 [21], compared with the expected location of case C mass transfer from population synthesis models (red ellipses) and merger products from population synthesis models (green polygons). As can be seen, the observed BSSs are consistent with being merger products (and since most of them are in binaries, their progenitors must be triples in this case), but, for the most, are inconsistent with being case C mass transfer products. Population synthesis models were made using the BSSE code (taken from the senior thesis of M. Bailey — adviser: R. Mathieu).

11.5.3 Field BSSs

11.5.3.1 Low Luminosity Halo BSSs

Comparing the various expectations and constraints from the currently suggested models for BSS formation, it appears that low luminosity field BSSs such as studied by [5] could be fully explained by case B and C MT. Most of these BSSs are found in binary systems with periods in the range 167 – 1576 day period, a typical ΔM in the range 0.03 – 0.48 M_{\odot} , with non-circular, but low eccentricity orbits. The binary companions are not seen, and are likely WDs; their minimal masses range between 0.18 – 0.55 M_{\odot} . Most of these properties are consistent with the predictions of

the case C MT, beside the BSSs with the largest ΔM which appear to be too high compared with theoretical predictions. These, however, might serve as evidence for additional formation route, either a case B MT or triple evolution scenario. In the case B MT, larger ΔM are possible. In this case we expect BSSs with the largest ΔM to have the lowest mass companions, corresponding to helium WDs expected in the case B MT. We might also expect to generally find them at shorter periods. One peculiarity is the relatively high eccentricities of these most massive BSS binaries, which is less clear in the context of MT. This might be explained by the triple secular evolution model, in which the BSS binary eccentricities are not expected to be low, the formed BSS are massive merger products, and the most typical companions are low mass MS companions. The observed periods, however, are shorter than seen for the relevant triples in the field (see Fig. 11.1). Unfortunately, the statistics are currently too few to confirm/refute the eccentricity-mass trend.

It therefore appears that at least in the case of low luminosity field BSSs our theoretical understanding of their origin from case C MT with a contribution from case B and/or triple evolution case is fully consistent with observations. Taking this path, we can now try and learn about case B/C MT from these observations. Though the observed orbital eccentricities are not zero, they are much lower than the typical eccentricities of binaries with similar periods in the field (with the mentioned caveat for the massive BSSs), showing that binaries are circularised, though at lower rates than expected from current theories. We will discuss this issue in the context of OC BSSs.

11.5.3.2 Other Field BSSs

Other populations of BSSs exist outside stellar clusters. Their characterisation, however, is still in an early stage, and will only be briefly mentioned here.

Luminous halo BSSs Current studies of halo BSSs focused on low luminosity BSSs, while luminous, likely more massive BSSs have hardly been studied. Massive main sequence stars do exist in the halo, as shown, for example, by the discoveries of hyper-velocity main sequence B-stars. Most of these stars are thought to be regular main sequence stars ejected at high velocities from the Galactic Centre, while a small fraction might have been ejected from dense young clusters. Some, however, appear too young to have had the time to propagate from such locations in the Galactic Centre, or from young stellar clusters in the Galactic disc, and might have been rejuvenated through mass transfer, making them BSSs [54]. Additional observations of halo BSSs are needed in order to characterise their overall population.

Bulge BSSs Recently, the first bulge BSSs have been detected: about a quarter of them appears to be in short period W UMa type binaries [9]. Their overall fraction compared to background HB population is consistent with that found for halo BSSs by [6]. The low densities in the bulge require the BSS formation to go through a binary/triple evolution interaction and not through collisions. From the large number of short period binaries, it appear that case A MT and mergers are likely the dominant channel. Given the likely origin of short period binaries in triples [14, 61, 15],

it is suggestive that the dominant route for BSS formation in the bulge is through triple evolution leading to mergers and case A MT [56].

11.6 Summary

Blue stragglers exist in a wide variety of environments, ranging from low density environments such as the Galactic halo and bulge through open clusters to dense globular clusters. The amount of observational data, varies widely from one environment to the other. The largest sets of data are available for GCs, however the most detailed data including specific properties of binary BSSs are of open clusters such as NGC 188 and M67, but the latter include only a small number of BSSs. The study of blue stragglers and their origin touches upon a wide range of fields, ranging from stellar evolution, stellar collisions, dynamics of few-body systems and the overall evolution of stellar clusters. Though the BSS phenomena exist in many environments, it is not yet clear whether the same processes play similar roles in their production. Current theories for the origin of BSSs include various type of mass transfer or merger products in binaries; possible direct collision origin; or induced merger/collision in secularly evolving triple systems.

The advance in complex simulation of clusters which include both dynamics and stellar evolution provide a wealth of theoretical predictions which could be compared for the first time with observations. Comparisons between the simulations and the observations suggest that the data on BSSs in GCs are consistent with the BSSs having a collisional origin mostly from binary-single and binary-binary encounters. However, given the major role of binary/triple systems in the theory of BSS formation but the lack of information on the binarity of BSSs, it is still premature to conclude that the BSS formation in GCs is well understood. Much more detailed data are available for OCs, but current simulations show striking failure in reproducing the observed populations. The currently most likely origin of BSSs in OCs is likely a combinations of induced mergers and case A mass transfer in triples as well as case C/D mass transfer in evolved binaries. It does appear clear that collisions play at most a minor role in producing OC BSSs and binary BSSs in these environments.

Field BSSs cannot form through collisions due to the low stellar density environments. It appears that the current data on galactic halo BSSs are consistent with case B/C mass transfer, though triple secular evolution may also contribute. Less data is available for bulge BSSs, but the large fraction of eclipsing binaries among them suggest that secular triples play an important role (producing the short period binaries, that then produce BSSs through case A mass transfer and mergers). Case B/C mass transfer are also likely to play a role in those cases.

We conclude that BSSs are likely to have multiple origins both different origins in different environments, as well as combinations of various evolutionary/dynamical channels in the any given environments.

Table 11.1 Summary of BSSs properties in different environments

Environment Property	GC	OC	Halo	Bulge
Inferred ΔM (M_{\odot})	$0.2 - 0.8$	$0.2 - 0.8$	$0.03 - 0.48$	
Frequency	$10^{-5} - 10^{-4}$	$\sim f_{ew} \times 10^{-2}$	$1/2000$	
Spatial distribution	Centrally concentrated and sometimes bi-model	Centrally concentrated and sometimes bi-model	–	–
Binarity	Unknown; Large fraction in eclipsing binaries	High; 76 % in NGC 188	High; consistent with 100% $0.18 \leq M_{min} \leq 0.55 M_{\odot}$	at least 25% are w UMa short period binaries
Companion mass	unknown	Peaks at $0.55 M_{\odot}$		unknown
Period	unknown	$P > 500$ days	$167 - 1576$ days	$> 25\% P < 10$ days
Eccentricity	unknown	$10\% P < 10$ days High; $\langle e \rangle \sim 0.34$	typical $200 - 800$ days Low; $\langle e \rangle \sim 0.17$	unknown
Environmental correlates	$f_{BSS} \propto M_{cluster}$ $f_{BSS} \propto f_{bin}$	too low statistics	–	–

Table 11.2 Summary of BSSs models and their predictions

Theoretical model	MT A / Merger	MT B	MTC	MT D	Collisions	Secular Triples	GC models w/o triples	OC models w/o triples
<i>Property</i>								
$\Delta M_{\max} (M_{\odot})$	turn-off mass	up to a few M_{\odot}	< 0.25	0.5 turn-off mass	turn-off mass	turn-off mass	turn-off mass	turn-off mass
Composition	regular MS	below turn-off mass	typical < 0.15	enriched AGB	and even higher	regular MS	combination of collision/merger	Mostly
Spatial distribution	—	regular MS	processed elements	processed elements	regular (?)	—	and MT products	MT/merger products
Binarity	0% (merger)	High 100 %	High 100 %	High 100 %	Centrally concentrated	High 100 %	Centrally concentrated; can be bimodal $\sim 60\%$	Intermediate
Companion	High 100 % (MT)	Helium WD	CO WD	CO WD	High (collisions in binaries)	Any type	Any type	Mostly CO WDs
Period	None (merger); MS (MT), likely w UMa type binary	$0.1 \lesssim M \lesssim 0.45$	$0.55 \lesssim M \lesssim 0.65$	$0.55 \lesssim M \lesssim 0.65$	Any type	Any type	Any type	Mostly large separations
Eccentricity	$P < 3 - 4$ days	typical	typical	typical	typical	> 500 days; inferred from field triples	typical	> 500 days
	Likely circular	$10 < P < 1000$	$300 < P < 2000$	$2000 < P < 10^4$	$10 < P < 1000$	typical over binaries of triples (ϵ) $\sim 0.3 - 0.4$	$10 < P < 1000$	Mostly circular
		Very low	Low	Somewhat low	High		High	(due to BSSE prescriptions ?)
		circularised	circularised					

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